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Investigating pedagogical content knowledge of high school teachers on mathematical proofs

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ABSTRACT

This study investigated the pedagogical content knowledge (PCK) of mathematics teachers regarding the teaching of mathematical proofs, focusing on three domains: knowledge of curriculum, knowledge of students, and knowledge of teaching strategies. Six mathematics teachers from Grades 7 to 10 and sixteen senior high school students from Tagana-an National High School in Surigao del Norte participated in the study. Employing a qualitative research design and grounded theory methodology, data were gathered through semi-structured interviews and a researcher-made proof test. Findings revealed that while teacher-respondents generally followed the sequence of lessons as presented in the learning materials and demonstrated familiarity with topics requiring proof, their implementation varied. Teachers identified common student difficulties such as lack of prior knowledge, comprehension issues, and challenges in reasoning and retention. To address these, demonstration and step-by-step explanation were the most commonly employed strategies. However, some teachers admitted to skipping proof-related lessons due to time constraints and difficulty in delivering the content effectively. Student-respondents, on the other hand, expressed limited understanding of proofs, associating them mainly with justification of solutions. Results of the proof test indicated that students did not meet the expected proficiency in mathematical reasoning, coherence, and clarity. Despite students' recognition of their teachers' efforts, a significant gap remained in proof writing performance. The study underscores the need for enhanced teacher training on proof instruction, greater emphasis on student thinking, and sustained curriculum support to improve the teaching and learning of mathematical proofs.

Keywords: pedagogical content knowledge, mathematical proofs, teacher knowledge, proof writing, qualitative research, secondary mathematics education

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INTRODUCTION

Mathematics is a subject that pervades life in all aspects. Its value goes beyond the classroom and the school. This means that mathematics as a subject must be taught comprehensively and with much depth. A number of factors may influence the teaching of mathematics but teachers play an important role in the teaching process.

Despite a teacher's deep understanding of a subject area he or she must also be able to foster understanding of subject or concepts for students. Shulman (1986) as cited by Bilash (2009) calls this pedagogical content knowledge (PCK). It refers to the overlapping of information about subject knowledge, that is knowledge of the subject being taught, and pedagogic knowledge, that is knowledge of how to teach.

Pedagogical content knowledge is grounded in the beliefs and practices of the teacher. It also includes conceptual and procedural knowledge, a repertoire of varied techniques or activities, knowledge of techniques for assessing and evaluating, and knowledge of a variety of resources which can be easily accessed for use in the classroom. Pedagogical content knowledge "represents a class of knowledge that is central to teachers' work and that would not typically be held by nonteaching subject matter experts or by teachers who know little of that subject" (Marks, 1990 cited by Bilash, 2009).

Reasoning and proof are central to the discipline of mathematics and hold a prominent place in the K to 12 Mathematics Curriculum. These are the skills and processes expected to be developed. In fact, Ross (1998) pointed that "the essence of mathematics lies in proofs". This is elaborated by Knuth (2002) when he said that "proof is not a thing separable from mathematics; it is an essential component of doing, communicating and recording mathematics."

Teaching and developing proof and reasoning skills for students depend on the teacher (Riley, 2004). The questions directed (Martino & Maher, 1999) and materials used in the classroom, along with teachers influence on the proving capacities of students (Stylianides, 2007). Besides, teachers' perceptions about proofs, experiences and skills are effective in the process of gaining proving skills by students (Knuth, 2002).

However, teaching mathematical proof is a challenging topic for teachers (Heinze & Reiss 2004). Many of the teachers hold limited views of the nature of proof in mathematics and demonstrated inadequate understanding of what constitutes proof (Knuth 2002). Mariotti (2006) added that the main difficulties encountered by most students have led many teachers to abandon this practice.

From these, the researchers would like to investigate the pedagogical content knowledge (PCK) of high school teachers on mathematical proofs.

Statement of the problem

This study aimed to investigate the pedagogical content knowledge of teachers of Tagana-an National High School on mathematical proofs. The PCK was investigated with respect to mathematics teachers' knowledge of curriculum, knowledge of students, and knowledge of teaching strategies. Furthermore, skills of students in proving are likely to be determined. Specifically, it sought to answers the following sub-questions:

1. How do teachers and students regard mathematical proof?
2. What is the nature of teachers' PCK regarding mathematical proofs?

3. What is the teachers' knowledge of curriculum regarding mathematical proof?
4. What is the teachers' knowledge of students regarding mathematical proofs?
5. What are the difficulties faced by students while learning mathematical proofs?
6. What strategies do teachers employ to improve students' skills in proof writing?
7. What strategies do teachers employ to overcome the difficulties students experience in proof writing?
8. What is the level of students' skills in proving?

METHODOLOGY

This study employed a qualitative research design to explore the Pedagogical Content Knowledge (PCK) of mathematics teachers in the context of teaching mathematical proofs. Given the complexity and context-dependent nature of PCK—which is shaped by teachers' experience, curriculum familiarity, and student characteristics—a qualitative approach was considered most appropriate (Aydemir, 2014). The study specifically adopted grounded theory methodology as proposed by Strauss and Corbin (1990), which allows for the development of theory grounded in systematically collected and analyzed data. Grounded theory avoids premature assumptions and instead allows patterns and meanings to emerge from the data itself, making it well-suited to study how PCK unfolds in authentic teaching environments.

The research was conducted at Tagana-an National High School, located in Barangay Sampaguita, Tagana-an, Surigao del Norte. The school offers both Junior and Senior High School programs, with 17 instructional classrooms and ongoing expansion for additional facilities. The teacher-respondents consisted of six mathematics teachers of varying ranks (Teacher I to Master Teacher I), teaching experiences (ranging from 2 to 30 years), and grade-level assignments (Grades 7 to 10). This range ensured diversity in teaching perspectives, particularly with regard to experience-based evolution of PCK.

Purposive sampling was used to select student-respondents, a method consistent with qualitative research goals of obtaining rich and in-depth information from individuals with relevant experiences. From a total of 162 Senior High School students enrolled in the academic year 2016–2017, 83 were identified to have completed their Junior High School in the same institution. These students were then categorized by proficiency level based on their Grade 10 final ratings, following DepEd Order No. 8, s. 2015. Four students from each level—Outstanding, Very Satisfactory, Satisfactory, and Unsatisfactory—were selected to ensure balanced representation. The selection also considered students' proximity to the school and willingness to participate in interviews conducted after class hours.

To collect data, the researchers utilized two primary instruments: semi-structured interviews and a proof test. The interviews were designed to investigate three key components of PCK: knowledge of curriculum, knowledge of student difficulties, and knowledge of teaching strategies. All interviews were conducted face-to-face, voice-recorded with consent, and guided by an interview protocol that allowed follow-up questions for clarification and depth. To complement the interviews and assess students' actual proficiency in mathematical proof writing, a 13-item proof test was administered. The test covered learning competencies from Grades 8 to 10, including theorems on triangle congruence, inequalities, similarity, and circle geometry. Responses were assessed using a rubric adapted from the Grade 8 Learning Materials, focusing on mathematical reasoning, coherence, and clarity.

The data gathering procedure involved securing the approval of school administrators and consent from all participants. Interviews and proof tests were conducted in a respectful and organized manner. The data analysis was carried out through the three stages of grounded theory: open coding, axial coding, and selective coding. During open coding, interview transcripts were broken into discrete segments, labeled with conceptual codes, and grouped into categories and subcategories based on shared meanings. Axial coding then reassembled the data by establishing relationships between these categories and validating them through cross-referencing with relevant literature. In selective coding, the researchers identified a core category that unified all other themes and developed an initial grounded theory that explained the phenomenon under investigation.

To enhance the reliability and validity of the findings, data triangulation was employed by integrating multiple data sources: teacher interviews, student interviews, and the results of the proof test. This triangulated approach allowed for a comprehensive understanding of PCK as demonstrated in both teacher discourse and student performance. Descriptive statistics—mean and standard deviation—were used to analyze the results of the proof test and to provide a quantitative description of students' proof writing skills in terms of reasoning, coherence, and clarity. Overall, the methodology ensured a rigorous, context-sensitive exploration of PCK that aligned with the research objectives and theoretical foundations of grounded qualitative inquiry.

FINDINGS AND DISCUSSION

Teachers' knowledge of curriculum about proofs

Teachers' knowledge of curriculum about proofs was obtained with the help of the PCK interview. The results of the study revealed that teachers' curriculum knowledge was mainly based on the learning materials. Their teachings were mainly based on the topics as sequenced in the textbook.

Grade 7 mathematics does not include learning competencies related to proof writing. However, when asked which lessons require proof, T1 mentioned Word Problems Involving Equalities. Both T1 and T3 answered that even though proofs are not formally introduced in Grade 7, students justifying their solution to problem were practiced in their classes. Grade 8 math teachers, T4 and T6 had identified all lessons that include proof writing. T3 as the grade 9 teacher only mentioned Similarity and forgot Quadrilaterals. T2 and T5 both knew that lessons on Circles include proof writing but forgot to mention The Remainder and Factor theorems.

Regarding teachers' understanding of proofs, teacher respondents defined proof as a way to verify mathematical results and justify truthful ness of a statement.

Teachers' Knowledge of Students about Proofs

In this section, teachers' knowledge about students' difficulties in learning mathematical proofs was summarized. Teachers' knowledge of students' difficulties in learning mathematical proofs was obtained through an interview.

Students' difficulties were categorized as to lack of required skills and poor background in mathematics. Teacher-respondents perceived that students have difficulty in analysis, comprehension and retention. This was supported with the responses of students. Students

admitted that they have difficulties in understanding the lesson, memorizing laws and properties, and in looking for ways to solve a problem.

Teachers' Knowledge of teaching strategies

Teacher-respondents followed the sequence of lessons as stated in the learning materials. Common strategy mentioned by the teachers was the demonstration approach wherein they explained the step by step procedures. According to the student-respondents, their teachers explained in detail and answered each student's questions while teaching. They provide examples, board work exercises, group activity and oral recitations. Students-respondents also mentioned the skills possessed by their teachers such as can explain the lesson well, teaching them short cuts, and provided motivation for them to learn better. However, few students mentioned that they encountered teachers who sometimes do not explain well which led to their confusion.

T2, T3, and T5 had discussed proof writing. However, T4 and T6 admitted that they were not able to cover all lessons provided in the materials, more importantly on the lessons that have activities on proof writing because they did not reach these topics.

Teachers admitted their difficulties in teaching proof such as formulating statements, providing reasons, connecting concepts, and selection of teaching strategy.

Other teaching activities mentioned by the teachers were journal writing and applying a reading program used by the Department of Education to address students' difficulties in proof writing.

Proficiency of students in proof writing

Student-respondents showed little knowledge of proofs. Proof to them, was only limited to justifying the correctness of a solution to a problem, providing evidences to make a statement true, supporting a theorem and validating an idea.

Proof writing skills of students were measured in terms of coherence, clarity and mathematical reasoning. Scores of the proof test significantly show poor results. Results reflected that students did not meet expectations in proof writing.

CONCLUSIONS

The findings revealed that the teacher-respondents generally adhered to the sequence of lessons as outlined in the learning materials. They recognized mathematical proof as a method for verifying results and justifying the truthfulness of statements. However, two teacher-respondents admitted to omitting the teaching of proof despite its presence in the curriculum, citing time constraints as the primary reason.

Challenges encountered in teaching proof included difficulties in formulating statements, providing justifications, connecting mathematical concepts, and selecting appropriate instructional strategies. The teacher-respondents were also aware of their students' struggles, noting that many lacked the foundational knowledge and necessary skills for effective proof writing. To address these challenges, most teachers employed demonstration as their primary teaching strategy.

While student-respondents acknowledged and appreciated their teachers' competence in teaching proofs, the assessment results indicated that senior high school students did not meet the

expected proficiency in proof writing, particularly in the areas of mathematical reasoning, clarity, and coherence.

Future Directions

In the light of the above findings and conclusions, the following recommendations are offered:

Teachers should be given opportunities to reflect on the specific topics on proof with regard to how to use knowledge of curriculum, knowledge of students and teaching strategies in order to make their teaching more effective.

Teachers are required to upgrade their knowledge and skills in proof writing. They should be exposed to seminars, training workshops and other skill's enhancement activities for teaching.

Teachers must try new ways of exploring how students are thinking and ways to develop their skills in proof writing.

Educational planners and researchers may conduct study and researches about the status of proofs in secondary schools.

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